# DYNAMIC ANALYSIS USING RESPONSE SPECTRUM SEISMIC LOADING

*Prior To The Existence Of Inexpensive Personal Computers The Response Spectrum Method Was The Standard Approach For Linear Seismic Analysis*

## 15.1 INTRODUCTION

The basic mode superposition method, which is restricted to linearly elastic analysis, produces the complete time history response of joint displacements and member forces. In the past there have been two major disadvantages in the use of this approach. First, the method produces a large amount of output information that can require a significant amount of computational effort to conduct all possible design checks as a function of time. Second, the analysis must be repeated for several different earthquake motions in order to assure that all frequencies are excited, since a response spectrum for one earthquake in a specified direction is not a smooth function.

There are computational advantages in using the response spectrum method of seismic analysis for prediction of displacements and member forces in structural systems. The method involves the calculation of only the maximum values of the displacements and member forces in each mode using smooth design spectra that are the average of several earthquake motions.

The purpose of this chapter is to summarize the fundamental equations used in the response spectrum method and to point out the many approximations and limitations of the method. For, example it cannot be used to approximate the nonlinear response of a complex three-dimensional structural system.

The recent increase in the speed of computers has made it practical to run many time history analyses in a short period of time. In addition, it is now possible to run design checks as a function of time, which produces superior results, since each member is not designed for maximum peak values as required by the response spectrum method.

## 15.2 DEFINITION OF A RESPONSE SPECTRUM

For three dimensional seismic motion, the typical modal Equation (13.6) is rewritten as

$$\ddot{y}(t)_n + 2\zeta_n \omega_n \dot{y}(t)_n + \omega_n^2 y(t)_n = p_{nx}\ddot{u}(t)_{gx} + p_{ny}\ddot{u}(t)_{gy} + p_{nz}\ddot{u}(t)_{gz} \tag{15.1}$$

where the three *Mode Participation Factors* are defined by $p_{ni} = -\phi_n^T \mathbf{M}_i$ in which i is equal to x, y or z. Two major problems must be solved in order to obtain an approximate response spectrum solution to this equation. First, for each direction of ground motion maximum peak forces and displacements must be estimated. Second, after the response for the three orthogonal directions is solved it is necessary to estimate the maximum response due to the three components of earthquake motion acting at the same time. This section will address the modal combination problem due to one component of motion only. The separate problem of combining the results from motion in three orthogonal directions will be discussed later in this chapter.

For input in one direction only, Equation (15.1) is written as

$$\ddot{y}(t)_n + 2\zeta_n \omega_n \dot{y}(t)_n + \omega_n^2 y(t)_n = p_{ni}\ddot{u}(t)_g \tag{15.2}$$

Given a specified ground motion $\ddot{u}(t)_g$, damping value and assuming $p_{ni} = -1.0$ it is possible to solve Equation (15.2) at various values of $\omega$ and plot a curve of the maximum peak response $y(\omega)_{MAX}$. For this acceleration input, the curve is by definition the ***displacement response spectrum*** for the earthquake motion. A different curve will exist for each different value of damping.

A plot of $\omega\, y(\omega)_{MAX}$ is defined as the ***pseudo-velocity spectrum*** and a plot of $\omega^2 y(\omega)_{MAX}$ is defined as the ***pseudo-acceleration spectrum***. These three curves are normally plotted as one curve on special log paper. However, these pseudo-values have minimum physical significance and are not an essential part of a response spectrum analysis. The true values for maximum velocity and acceleration must be calculated from the solution of Equation (15.2).

There is a mathematical relationship, however, between the pseudo-acceleration spectrum and the total acceleration spectrum. The total acceleration of the unit mass, single degree-of-freedom system, governed by Equation (15.2), is given by

$$\ddot{u}(t)_T = \ddot{y}(t) + \ddot{u}(t)_g \tag{15.3}$$

Equation (15.2) can be solved for $\ddot{y}(t)$ and substituted into Equation (15.3) which yields

$$\ddot{u}(t)_T = -\omega^2 y(t) - 2\xi\omega\dot{y}(t) \tag{15.4}$$

Therefore, for the special case of zero damping, the total acceleration of the system is equal to $\omega^2 y(t)$. For this reason, the ***displacement response spectrum*** curve is normally not plotted as modal displacement $y(\omega)_{MAX}$ vs $\omega$. It is standard to present the curve in terms of $S(\omega)$ vs. a period $T$ in seconds. where

$$S(\omega)_a = \omega^2 y(\omega)_{MAX} \quad \text{and} \quad T = \frac{2\pi}{\omega} \qquad \text{(15.5a) and (15.5b)}$$

The pseudo-acceleration spectrum, $S(\omega)_a$, curve has the units of acceleration vs. period which has some physical significance for zero damping only. It is apparent that all response spectrum curves represent the properties of the earthquake at a specific site and are not a function of the properties of the structural system. After an estimation is made of the linear viscous damping properties of the structure, a specific response spectrum curve is selected.

## 15.3 CALCULATION OF MODAL RESPONSE

The maximum modal displacement, for a structural model, can now be calculated for a typical mode $n$ with period $T_n$ and corresponding spectrum response value $S(\omega_n)$. The maximum modal response associated with period $T_n$ is given by

$$y(T_n)_{MAX} = \frac{S(\omega_n)}{\omega_n^2} \tag{15.6}$$

The maximum modal displacement response of the structural model is calculated from

$$\mathbf{u}_n = y(T_n)_{MAX}\,\phi_n \tag{15.7}$$

The corresponding internal modal forces, $f_{kn}$, are calculated from standard matrix structural analysis using the same equations as required in static analysis.

## 15.4 TYPICAL RESPONSE SPECTRUM CURVES

A ten second segment of the Loma Prieta earthquake motions, recorded on a soft site in the San Francisco Bay Area, is shown in Figure 15.1. The record has been corrected, by use of an iterative algorithm, for zero displacement, velocity and acceleration at the beginning and end of the ten second record. For the earthquake motions given in Figure 15.1a, the response spectrum curves for displacement and pseudo-acceleration are summarized in Figure 15.2a and 15.2b

The velocity curves have been intentionally omitted since they are not an essential part of the response spectrum method. Furthermore, it would require considerable space to clearly define terms such as peak ground velocity, pseudo velocity spectrum, relative velocity spectrum and absolute velocity spectrum.

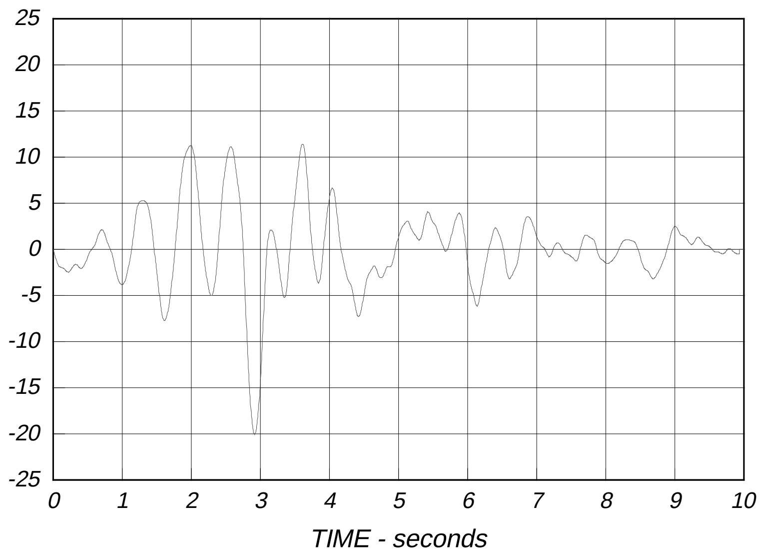



***Figure 15.1a. Typical Earthquake Ground Acceleration - Percent of Gravity***

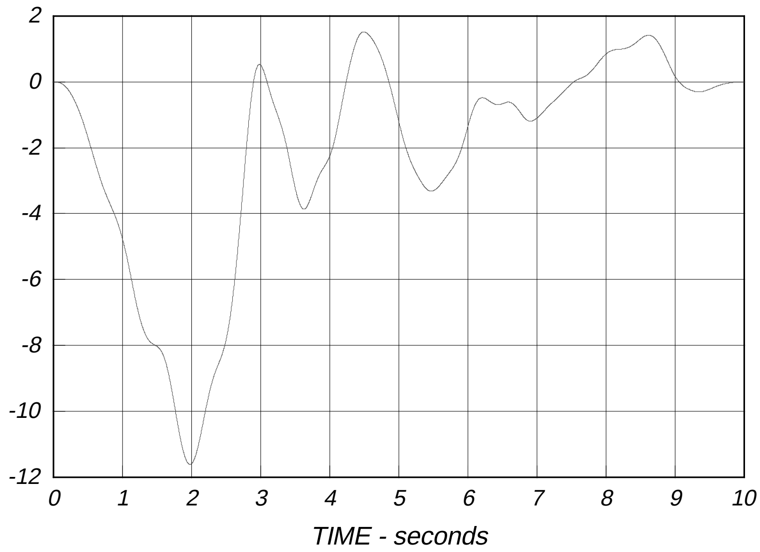



***Figure 15.1b. Typical Earthquake Ground Displacements - Inches***

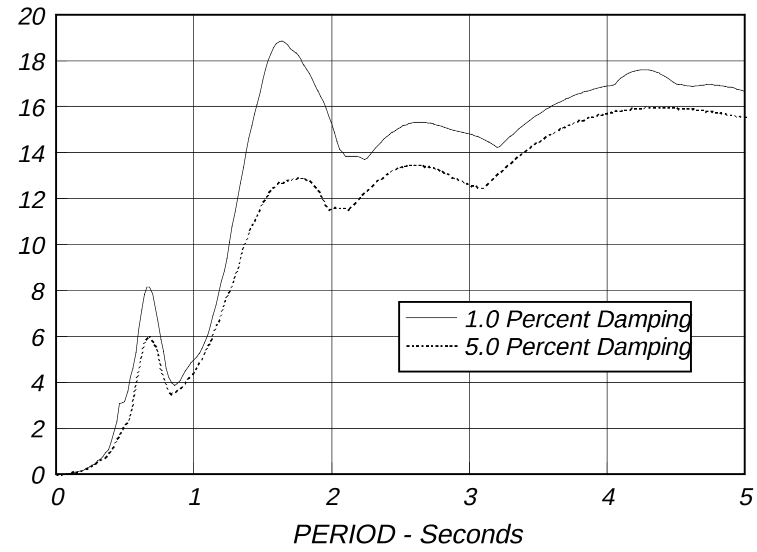



***Figure 15.2a. Relative Displacement Spectrum*** $y(\omega)_{MAX}$ ***- Inches***

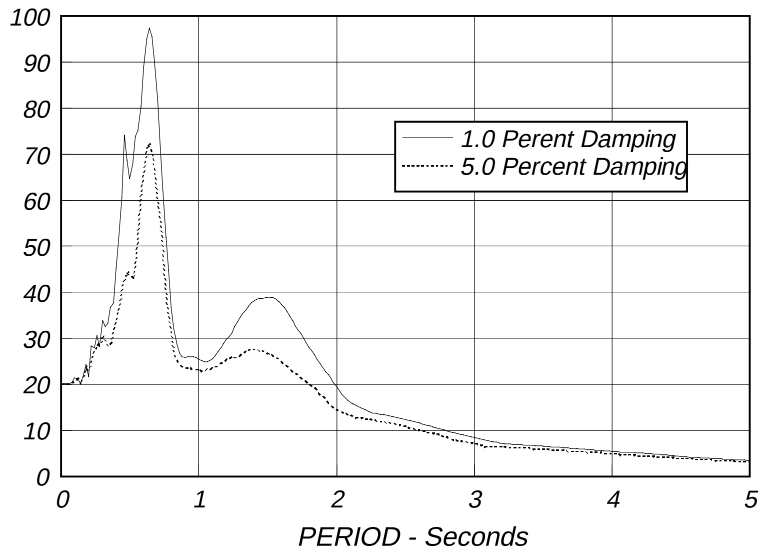



***Figure 15.2b. Pseudo Acceleration Spectrum,*** $S_a = \omega^2 y(\omega)_{MAX}$ ***- Percent of Gravity***

The maximum ground acceleration, for the earthquake defined by Figure 15.1a, is 20.01 percent of gravity at 2.92 seconds. It is important to note that the pseudo acceleration spectrum, shown in Figure 15.2b, has the same value for a very short period system. This is due to the physical fact that a very rigid structure moves as a rigid body and the relative displacements within the structure are equal to zero as indicated by Figure 15.2a. Also, the behavior of a rigid structure is not a function of the viscous damping value.

The maximum ground displacement shown in Figure 15.1b is -11.62 inches at 1.97 seconds. For long period systems, the mass of the one-degree-of-freedom structure does not move significantly and has approximately zero absolute displacement. Therefore, the relative displacement spectrum curves, shown in Figure 15.2a, will converge to 11.62 inches for long periods and all values of damping. This type of real physical behavior is fundamental to the design of base isolated structures.

The relative displacement spectrum, Figure 15.2a, and the absolute acceleration spectrum, Figure 15.2b, have physical significance. However, the maximum relative displacement is directly proportional to the maximum forces developed in the structure. For this earthquake the maximum relative displacement is 18.9 inches at a period of 1.6 seconds for one percent damping and 16.0 inches at a period of four seconds for five percent damping. It is important to note the significant difference between one and five percent damping for this typical soft site record.

Figure 15.2b, the absolute acceleration spectrum, indicates maximum values at a period of 0.64 seconds for both values of damping. Also, the multiplication by $\omega^2$ tends to completely eliminate the information contained in the long period range. Since most structural failures, during recent earthquakes, have been associated with soft sites, perhaps we should consider using the relative displacement spectrum as the fundamental form for selecting a design earthquake. The high frequency, short period, part of the curve should always be defined by

$$y(\omega)_{MAX} = \ddot{u}_{g\,MAX} / \omega^2 \qquad \text{or} \qquad y(T)_{MAX} = \ddot{u}_{g\,MAX} \frac{T^2}{4\pi^2} \tag{15.8}$$

where $\ddot{u}_{g\,MAX}$ is the peak ground acceleration.

## 15.5 THE CQC METHOD OF MODAL COMBINATION

The most conservative method that is used to estimate a peak value of displacement or force within a structure is to use the sum of the absolute of the modal response values. This approach assumes that the maximum modal values, for all modes, occur at the same point in time.

Another very common approach is to use the Square Root of the Sum of the Squares, SRSS, on the maximum modal values in order to estimate the values of displacement or forces. The SRSS method assumes that all of the maximum modal values are statistically independent. For three dimensional structures, in which a large number of frequencies are almost identical, this assumption is not justified.

The relatively new method of modal combination is the Complete Quadratic Combination, CQC, method [2] that was first published in 1981. It is based on random vibration theories and has found wide acceptance by most engineers and has been incorporated as an option in most modern computer programs for seismic analysis. Because many engineers and building codes are not requiring the use of the CQC method, one purpose of this chapter is to explain by example the advantages of using the CQC method and illustrate the potential problems in the use of the SRSS method of modal combination.

The peak value of a typical force can now be estimated, from the maximum modal values, by the CQC method with the application of the following double summation equation:

$$F = \sqrt{\sum_n \sum_m f_n \rho_{nm} f_m} \tag{15.9}$$

where $f_n$ is the modal force associated with mode $n$. The double summation is conducted over all modes. Similar equations can be applied to node displacements, relative displacements and base shears and overturning moments.

The cross-modal coefficients, $\rho_{nm}$, for the CQC method with constant damping are

$$\rho_{nm} = \frac{8\zeta^2 (1+r)\, r^{3/2}}{(1-r^2)^2 + 4\zeta^2 r(1+r)^2} \tag{15.10}$$

where $r = \omega_n / \omega_m$ and must be equal to or less than 1.0. It is important to note that the cross-modal coefficient array is symmetric and all terms are positive.

## 15.6 NUMERICAL EXAMPLE OF MODAL COMBINATION

The problems associated with the use of the absolute sum and the SRSS of modal combination can be illustrated by their application to the four story building shown in Figure 15.3. The building is symmetrical; however, the center of mass, of all floors, is located 25 inches from the geometric center of the building.

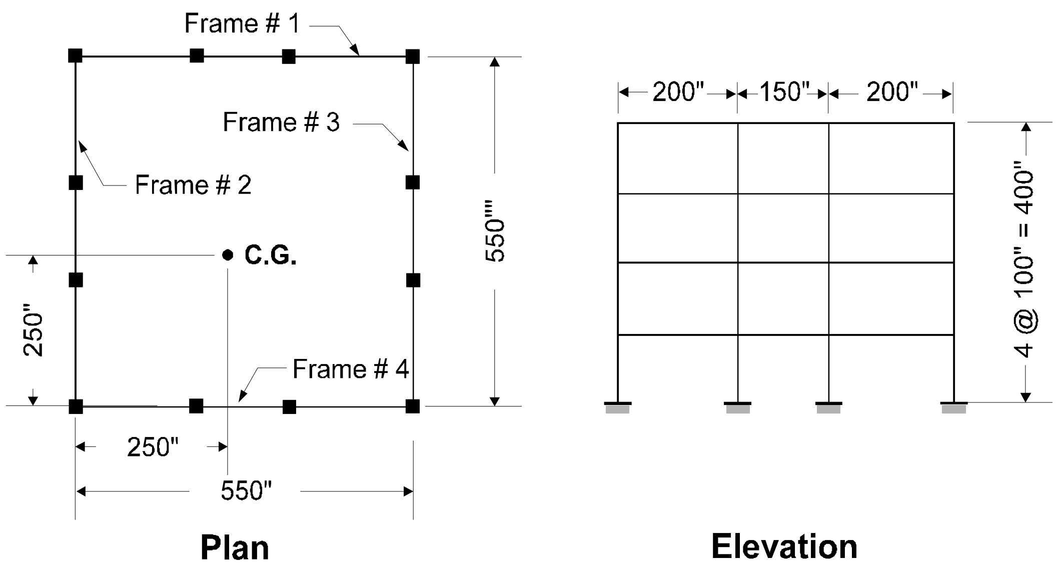
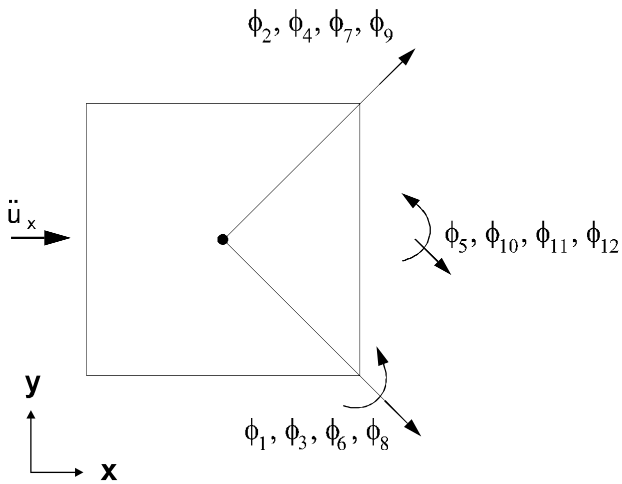



*Figure 15.3. A Simple Three Dimensional Building Example*




| Mode | Frequencies (Radians/sec.) |
|---|---|
| 1 | 13.869 |
| 2 | 13.931 |
| 3 | 43.995 |
| 4 | 44.189 |
| 5 | 54.418 |
| 6 | 77.686 |
| 7 | 78.029 |
| 8 | 108.32 |
| 9 | 108.80 |
| 10 | 172.613 |
| 11 | 304.80 |
| 12 | 425.00 |

***Figure 15.4 Frequencies and Approximate Directions of Mode Shapes***

The direction of the applied earthquake motion, a table of natural frequencies and the principal direction of the mode shape are summarized in Figure 15.4. One notes the closeness of the frequencies which is typical of most three dimensional building structures that are designed to equally resist earthquakes from both directions. Because of the small mass eccentricity, which is normal in real structures, the fundamental mode shape has x, y, as well as torsion components. Therefore, the model represents a very common three dimensional building system. Also, note that there is not ***a mode shape in a particular given direction*** as implied in many building codes and some text books on elementary dynamics.

The building was subjected to one component of the Taft, 1952, earthquake. An exact time history analysis, using all 12 modes, and a response spectrum analysis were conducted. The maximum modal base shears in the four frames for the first five modes are shown in Figure 15.5.

Figure 15.6 summarizes the maximum base shears, in each of the four frames, using different methods. The time history base shears, Figure 15.6a, are exact. The SRSS method, Figure 15.6b, produces base shears which under-estimate the exact values in the direction of the loads by approximately 30 percent and over-estimate the base shears normal to the loads by a factor of ten. The sum of the absolute values, Figure 15.6c, grossly over-estimates all results. The CQC method, Figure 15.6d, produces very realistic values that are close to the exact time history solution.

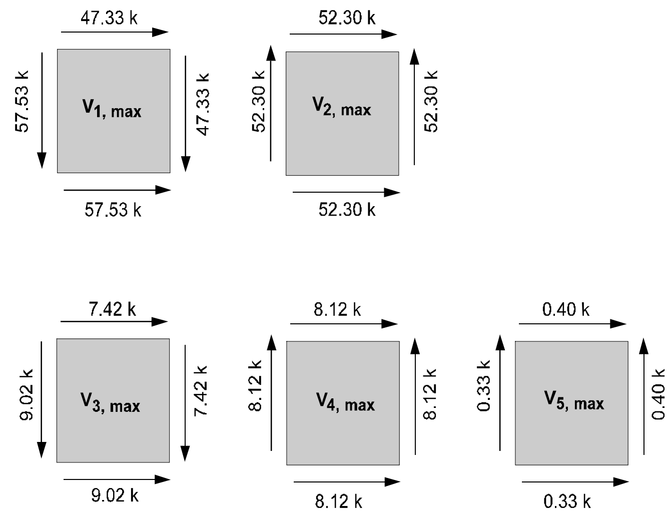



*Figure 15.5. Base Shears in Each Frame for First Five Modes*

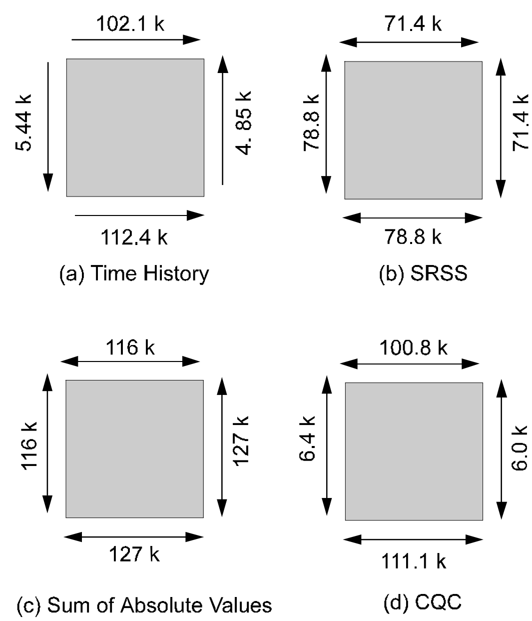



*Figure 15.6. Comparison of Modal Combination Methods*

The modal cross-correlation coefficients for this building are summarized in Table 15.1. It is of importance to note the existence of the relatively large off-diagonal terms that indicate which modes are coupled.

**Table 15.1. Modal Cross-Correlation Coefficients -** $\zeta = 0.05$

| **Mode** | 1 | 2 | 3 | 4 | 5 | $\omega_n$ **rad/sec** |
|---|---|---|---|---|---|---|
| 1 | 1.000 | 0.998 | 0.006 | 0.006 | 0.004 | 13.87 |
| 2 | 0.998 | 1.000 | 0.006 | 0.006 | 0.004 | 13.93 |
| 3 | 0.006 | 0.006 | 1.000 | 0.998 | 0.180 | 43.99 |
| 4 | 0.006 | 0.006 | 0.998 | 1.000 | 0.186 | 44.19 |
| 5 | 0.004 | 0.004 | 0.180 | 0.186 | 1.000 | 54.42 |

If one notes the signs of the modal base shears, shown in Figure 15.3, it is apparent how the application of the CQC method allows the sum of the base shears in the direction of the external motion to be added directly. In addition, the sum of the base shears, normal to the external motion, tend to cancel. The ability of the CQC method to recognize the relative sign of the terms in the modal response is the key to the elimination of errors in the SRSS method.

## 15.7 DESIGN SPECTRA

Design spectra are not uneven curves as shown in Figure 15.2 since they are intended to be the average of many earthquakes. At the present time, many building codes specify design spectra in the form shown in Figure 15.7.

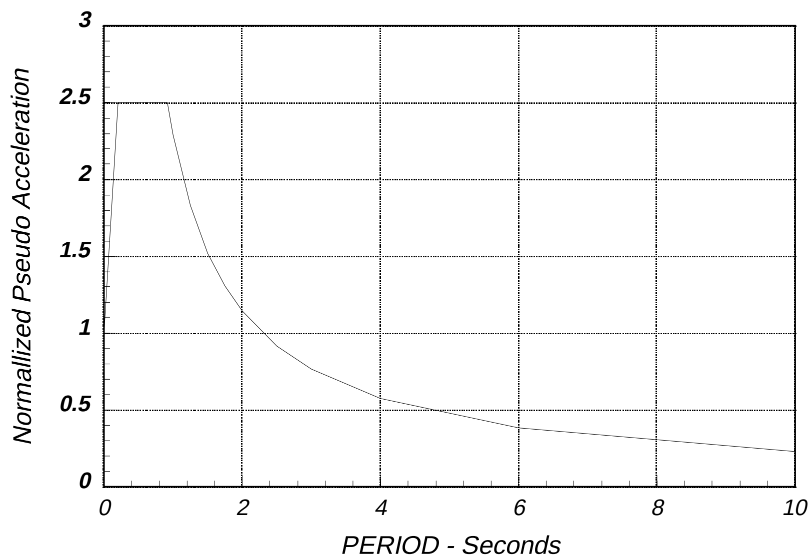



***Figure 15.7 Typical Design Spectrum***

The Uniform Building Code has defined specific equations for each range of the spectrum curve for four different soil types. For major structures it is now common practice to develop a site-dependent design spectrum which includes the effect of local soil conditions and distance to the nearest faults.

## 15.8 ORTHOGONAL EFFECTS IN SPECTRAL ANALYSIS

A well-designed structure should be capable of equally resisting earthquake motions from all possible directions. One option in existing design codes for buildings and bridges requires that members be designed for "100 percent of the prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction". Other codes and organizations require the use of 40 percent rather than 30 percent. However, they give no indication on how the directions are to be determined for complex structures. For structures that are rectangular and have clearly defined principal directions, these "percentage" rules yield approximately the same results as the SRSS method.

For complex three dimensional structures such as non-rectangular buildings, curved bridges, arch dams or piping systems, the direction of the earthquake which produces the maximum stresses, in a particular member or at a specified point, is not apparent. For time history input, it is possible to perform a large number of dynamic analyses at various angles of input in order to check all points for the critical earthquake directions. Such an elaborate study could conceivably produce a different critical input direction for each stress evaluated. However, the cost of such a study would be prohibitive.

It is reasonable to assume that motions that take place during an earthquake have one principal direction [1]. Or, during a finite period of time, when maximum ground acceleration occurs, a principal direction exists. For most structures this direction is not known and, for most geographical locations, cannot be estimated. Therefore, the only rational earthquake design criterion is that the structure must resist an earthquake of a given magnitude from any possible direction. In addition to the motion in the principal direction, a probability exists that motions normal to that direction will occur simultaneously. In addition, because of the complex nature of three dimensional wave propagation, it is valid to assume that these normal motions are statistically independent.

Based on these assumptions, a statement of the design criterion is "a structure must resist a major earthquake motion of magnitude $S_1$ for all possible angles $\theta$ and, at the same point in time, resist earthquake motions of magnitude $S_2$ at $90^o$ to the angle $\theta$". These motions are shown schematically in Figure 15.1.

### 15.8.1 Basic Equations For Calculation Of Spectral Forces

The stated design criterion implies that a large number of different analyses must be conducted in order to determine the maximum design forces and stresses. It will be shown, in this section, that maximum values for all members can be exactly evaluated from one computer run in which two global dynamic motions are applied. Furthermore, the maximum member forces calculated are invariant with respect to the selection system.

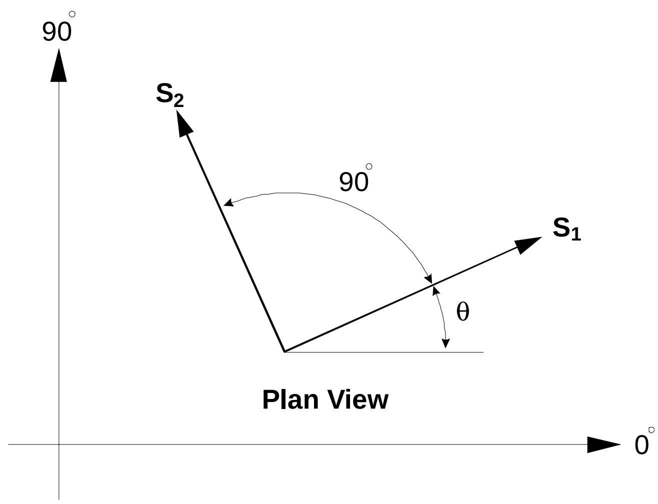



***Figure 15.7. Definition of Earthquake Spectra Input***

Figure 15.7 indicates that the basic input spectra $S_1$ and $S_2$ are applied at an arbitrary angle $\theta$. At some typical point within the structure, a force, stress or displacement $F$ is produced by this input. In order to simplify the analysis, it will be assumed that the minor input spectrum is some fraction of the major input spectrum. Or,

$$S_2 = a S_1 \tag{15.11}$$

where $a$ is a number between 0 and 1.0.

Recently, Menun and Der Kiureghian [3] presented the CQC3 method for the combination of the effects of orthogonal spectrum.

The fundamental CQC3 equation for the estimation of a peak value is

$$F = [F_0^2 + a^2 F_{90}^2 - (1-a^2)(F_0^2 - F_{90}^2)\sin^2\theta + 2(1-a^2)F_{0-90}\sin\theta\cos\theta + F_z^2]^{\frac{1}{2}} \tag{15.12}$$

where

$$F_0^2 = \sum_n \sum_m f_{0n}\rho_{nm} f_{0m} \tag{15.13}$$

$$F_{90}^2 = \sum_n \sum_m f_{90n}\rho_{nm} f_{90m} \tag{15.14}$$

$$F_{0-90} = \sum_n \sum_m f_{0n}\rho_{nm} f_{90m} \tag{15.15}$$

$$F_Z^2 = \sum_n \sum_m f_{zn}\rho_{nm} f_{zm} \tag{15.16}$$

in which $f_{0n}$ and $f_{90n}$ are the modal values produced by 100 percent of the lateral spectrum applied at 0 and 90 degrees respectively and $f_{zn}$ is the modal response from the vertical spectrum which can be different from the lateral spectrum.

It is important to note that for equal spectra $a = 1$, the value $F$ is not a function of $\theta$ and the selection of the analysis reference system is arbitrary. Or,

$$F_{MAX} = \sqrt{F_0^2 + F_{90}^2 + F_z^2} \tag{15.17}$$

This indicates that it is possible to conduct only one analysis, with any reference system, and the resulting structure will have all members that are designed to equally resist earthquake motions from all possible directions. This method is acceptable by most building codes.

### 15.8.2 The General CQC3 Method

For a=1 the CQC3 method reduces to the SRSS method. However, this can be over conservative since real ground motions of equal value in all directions have not been recorded. Normally, the value of $\theta$ in Equation (15.12) is not known; therefore, it is necessary to calculate the critical angle that produces the maximum response. Differentiation of Equation (15.12) and setting the results to zero yields

$$\theta_{cr} = \frac{1}{2}\tan^{-1}[\frac{2F_{0-90}}{F_0^2 - F_{90}^2}] \tag{15.17}$$

Two roots exist for Equation (15.17) that must be checked in order that the following equation is maximum:

$$\begin{aligned} F_{MAX} = [F_0^2 + a^2 F_{90}^2 - (1-a^2)(F_0^2 - F_{90}^2)\sin^2\theta_{cr} \\ - 2(1-a^2)F_{0-90}\sin\theta_{cr}\cos\theta_{cr} + F_z^2]^{\frac{1}{2}} \end{aligned} \tag{15.18}$$

At the present time no specific guidelines have been suggested for the value of $a$. Reference [3] presented an example with values $a$ between 0.50 and 0.85.

### 15.8.3 Examples Of Three Dimensional Spectra Analyses

The previously presented theory clearly indicates that the CQC3 combination rule, with $a$ equal 1.0, is identical to the SRSS method and produces results, for all structural systems, which are not a function of the reference system used by the engineer. One example will be presented in order to show the advantages of the method. A very simple one-story structure, shown in Figure 15.8, was selected to compare the results of the 100/30 and 100/40 percentage rules with the SRSS rule. Note that the masses are not at the geometric center of the structure. The structure has two translations and one rotational degrees-of-freedom located at the center of mass. The columns, which are subjected to bending about the local 2 and 3 axes, are pinned at the top where they are connected to an in-plane rigid diaphragm.

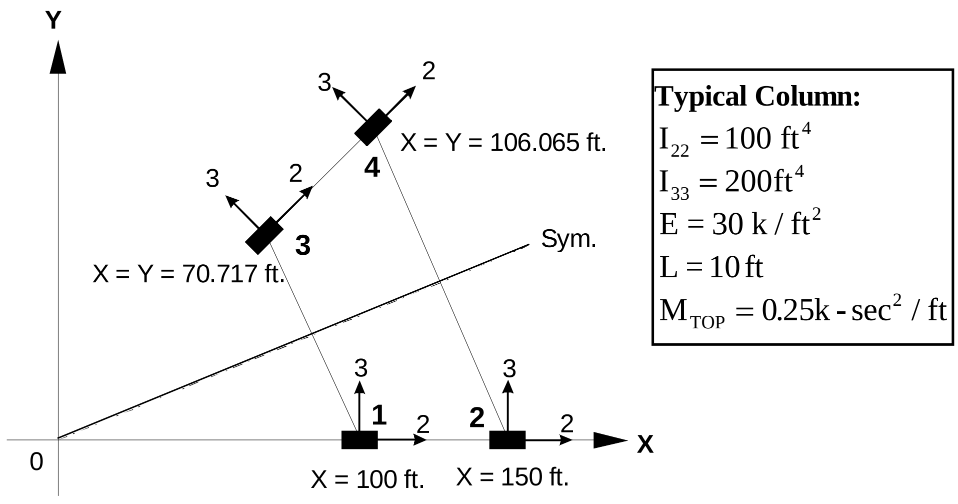



***Figure 15.8. Three Dimensional Structure***

The periods and normalized base shear forces associated with the mode shapes are summarized in Table 15.2. Since the structure has a plane of symmetry at 22.5 degrees, the second mode has no torsion and has a normalized base shear at 22.5 degrees with the x-axis. Due to this symmetry, it is apparent that columns 1 and 3 (or columns 2 and 4) should be designed for the same forces.

**Table 15.2. Periods and Base Reaction Forces**

| Mode | Period Seconds | X-Force | Y-Force | X-Moment | Y-Moment | Torsion |
|---|---|---|---|---|---|---|
| 1 | 1.01029 | .383 | -.924 | 9.24 | 3.83 | -115.5 |
| 2 | 0.76918 | -.924 | -.383 | 3.83 | -9.24 | 0.0 |
| 3 | 0.43102 | .383 | -.924 | 9.24 | 3.83 | -115.5 |

The definition of the mean displacement response spectrum used in the spectra analysis is given in Table 15.3.

**Table 15.3. Participating Masses and Response Spectrum Used**

| Mode | Period Seconds | X-MASS | Y-MASS | Spectral Displacement Used For Analysis |
|---|---|---|---|---|
| 1 | 1.01029 | 14.43 | 84.081 | 1.00 |
| 2 | 0.76918 | 85.36 | 14.65 | 1.00 |
| 3 | 0.43102 | 0.22 | 1.27 | 1.00 |

The moments about the local 2 and 3 axes at the base of each of the four columns for the spectrum applied separately at 0.0 and 90 degrees are summarized in Tables 15.4 and 15.5 and are compared to the 100/30 rule.

**Table 15.4. Moments About 2-Axes - 100/30 Rule**

| Member | $M_0$ | $M_{90}$ | $M_{SRSS} = \sqrt{M_0^2 + M_{90}^2}$ | $M_{100/30}$ | Error% |
|---|---|---|---|---|---|
| 1 | 0.742 | 1.750 | 1.901 | 1.973 | 3.8 |
| 2 | 1.113 | 2.463 | 2.703 | 2.797 | 3.5 |
| 3 | 0.940 | 1.652 | 1.901 | 1.934 | 1.8 |
| 4 | 1.131 | 2.455 | 2.703 | 2.794 | 3.4 |

**Table 15.5. Moments About 3-Axes - 100/30 Rule**

| Member | $M_0$ | $M_{90}$ | $M_{SRSS} = \sqrt{M_0^2 + M_{90}^2}$ | $M_{100/30}$ | Error% |
|---|---|---|---|---|---|
| 1 | 2.702 | 0.137 | 2.705 | 2.743 | 1.4 |
| 2 | 2.702 | 0.137 | 2.705 | 2.743 | 1.4 |
| 3 | 1.904 | 1.922 | 2.705 | 2.493 | -7.8 |
| 4 | 1.904 | 1.922 | 2.705 | 2.493 | -7.8 |

For this example, the maximum forces do not vary significantly between the two methods. However, it does illustrate that the 100/30 combination method produces moments which are not symmetric, whereas the SRSS combination method produces logical and symmetric moments. For example, member 4 would be over-designed by 3.4 percent about the local 2-axis and under-designed by 7.8 percent about the local 3-axis using the 100/30 combination rule.

The SRSS and 100/40 design moments about the local 2 and 3 axes at the base of each of the four columns are summarized in Tables 15.6 and 15.7

**Table 15.6. Moments About 2-Axes - 100/40 Rule**

| Member | $M_0$ | $M_{90}$ | $M_{SRSS} = \sqrt{M_0^2 + M_{90}^2}$ | $M_{100/40}$ | Error% |
|---|---|---|---|---|---|
| 1 | 0.742 | 1.750 | 1.901 | 2.047 | 7.7 |
| 2 | 1.113 | 2.463 | 2.703 | 2.908 | 7.6 |
| 3 | 0.940 | 1.652 | 1.901 | 2.028 | 1.2 |
| 4 | 1.131 | 2.455 | 2.703 | 2.907 | 7.5 |

**Table 15.7. Moments About 3-Axes - 100/40 Rule**

| Member | $M_0$ | $M_{90}$ | $M_{SRSS} = \sqrt{M_0^2 + M_{90}^2}$ | $M_{100/40}$ | Error% |
|---|---|---|---|---|---|
| 1 | 2.702 | 0.137 | 2.705 | 2.757 | 1.9 |
| 2 | 2.702 | 0.137 | 2.705 | 2.757 | 1.9 |
| 3 | 1.904 | 1.922 | 2.705 | 2.684 | -0.8 |
| 4 | 1.904 | 1.922 | 2.705 | 2.684 | -0.8 |

The results presented in Tables 15.6 and 15.7 also illustrate that the 100/40 combination method produces results which are not reasonable. Because of symmetry, members 1 and 3 and members 2 and 4 should be designed for the same moments. Both the 100/30 and 100/40 rules fail this simple test.

If a structural engineer wants to be conservative, the results of the SRSS directional combination rule or the input spectra can be multiplied by an additional factor greater than one. One should not try to justify the use of the 100/40 percentage rule because it is conservative in "most cases". For complex three dimensional structures the use of the 100/40 or 100/30 percentage rule will produce member designs which are not equally resistant to earthquake motions from all possible directions.

### 15.8.4 Recommendations On Orthogonal Effects

For three dimensional response spectra analyses, it has been shown that the "design of elements for 100 percent of the prescribed seismic forces in one direction plus 30 or 40 percent of the prescribed forces applied in the perpendicular direction" is dependent on the user's selection of the reference system. These commonly used "percentage combination rules" are empirical and can underestimate the design forces in certain members and produce a member design which is relatively weak in one direction. It has been shown that the alternate building code approved method, in which an SRSS combination of two 100 percent spectra analyses with respect to any user defined orthogonal axes, will produce design forces that are not a function of the reference system. Therefore, the resulting structural design has equal resistance to seismic motions from all directions.

The use of the CQC3 method should be used if a value of $a$ less than 1.0 can be justified. It will produce realistic results that are not a function of the user selected reference system.

## 15.9 LIMITATIONS OF THE RESPONSE SPECTRUM METHOD

It is apparent that use of the response spectrum method has limitations, some of which can be removed by additional development. However, it will never be accurate for nonlinear analysis of multi-degree of freedom structures. The author believes that in the future more time history dynamic response analyses will be conducted and the many approximations associated with the use of the response spectrum method will be avoided. Some of these additional limitations will be discussed in this section.

### 15.9.1 Story Drift Calculations

All displacements produced by the response spectrum method are positive numbers. Therefore, a plot of a dynamic displaced shape has very little meaning since each displacement is an estimation of the maximum value. Inter-story displacements are used to estimate damage to nonstructural elements and cannot be calculated directly from the probable peak values of displacement. A simple method to obtain a probable peak value of shear strain is to place a very thin panel element, with a shear modulus of unity, in the area where the deformation is to be calculated. The peak value of shear stress will be a good estimation of the damage index. The current code suggests a maximum value of 0.005 horizontal drift ratio, which is the same as panel shear strain if the vertical displacements are neglected.

### 15.9.2 Estimation of Spectra Stresses in Beams

The fundamental equation for the calculation of the stresses within the cross section of a beam is

$$\sigma = \frac{P}{A} + \frac{M_y x}{I_y} + \frac{M_x y}{I_x} \tag{15.19}$$

This equation can be evaluated for a specified x and y point in the cross section and for the calculated maximum spectral axial force and moments which are all positive values. It is apparent that the resulting stress may be conservative since all forces will probably not obtain their peak values at the same time.

For response spectrum analysis, the correct and accurate approach for the evaluation of equation (15.19) is to evaluate the equation for each mode of vibration. This will take into consideration the relative signs of axial forces and moments in each mode. An accurate value of the maximum stress can then be calculated from the modal stresses using the CQC double sum method. It has been the author's experience, with large three dimensional structures, that stresses calculated from modal stresses can be less than 50 percent of the value calculated using maximum peak values of moments and axial force.

#### 15.9.3 Design Checks for Steel and Concrete Beams

Unfortunately, most design check equations for steel structures are written in terms of "design strength ratios" which are a nonlinear function of the axial force in the member; therefore, the ratios cannot be calculated in each mode. A new approximate method, to replace the current state of the art approach of calculating strength ratios based on maximum peak values of member forces, is proposed by the author. This would involve first calculating the maximum axial force. The design ratios would then be evaluated mode by mode, assuming the maximum axial force reduction factor remains constant for all modes. The design ratio for the member would then be estimated by a double-sum modal combination method such as the CQC3 method. This approach would improve accuracy and still be conservative.

For concrete structures additional development work is required in order to develop a completely rational method for the use of maximum spectral forces in a design check equation because of the nonlinear behavior of concrete members. A time history analysis may be the only approach that will produce rational design forces.

#### 15.9.4 Calculation of Shear Force in Bolts

With respect to the interesting problem of calculating the maximum shear force in a bolt, it is not correct to estimate the maximum shear force from a vector summation since the x and y shears do not obtain their peak values at the same time. A correct method of estimating the maximum shear in a bolt is to check the maximum bolt shear at several different angles about the bolt axis. This would be a tedious approach using hand calculations; however, if the approach is built into a post processor computer program, the computational time to calculate the maximum bolt force is trivial.

The same problem exists if principal stresses are to be calculated from a response spectrum analysis. One must check at several angles in order to estimate the maximum and minimum value of the stress at each point in the structure.

## 15.10 SUMMARY

In this chapter it has been illustrated that the response spectrum method of dynamic analysis must be used carefully. The CQC method should be used to combine modal maxima in order to minimize the introduction of avoidable errors. The increase in computational effort, as compared to the SRSS method, is small compared to the total computer time for a seismic analysis. The CQC method has a sound theoretical basis and has been accepted by most experts in earthquake engineering. The use of the absolute sum or the SRSS method for modal combination cannot be justified.

In order for a structure to have equal resistance to earthquake motions from all directions, the CQC3 method should be used to combine the effects of earthquake spectra applied in three dimensions. The percentage rule methods have no theoretical basis and are not invariant with respect to the reference system.

Engineers, however, should clearly understand that the response spectrum method is an approximate method used to estimate maximum peak values of displacements and forces and that it has significant limitations. It is restricted to linear elastic analysis in which the damping properties can only be estimated with a low degree of confidence. The use of nonlinear spectra, which are commonly used, has very little theoretical background and should not be used for the analysis of complex three dimensional structures. For such structures, true nonlinear time-history response should be used as indicated in Chapter 19.

## 15.11 REFERENCES

1. J. Penzien and M. Watabe, "Characteristics of 3-D Earthquake Ground Motions," *Earthquake Engineering and Structural Dynamics*, Vol. 3, pp. 365-373, 1975.

2. E. L. Wilson, A. Der Kiureghian and E. R. Bayo, "A Replacement for the SRSS Method in Seismic Analysis," *Earthquake Engineering and Structural Dynamics*, Vol. 9, pp. l87-l92, 1981.

3. C. Menun and A. Der Kiureghian, “A Replacement for the 30 % Rule for Muticomponent Excitation”, *Earthquake Spectra, Vol. 13*, Number 1, February 1998.